



**DECSAI**

**Departamento de Ciencias de la Computación e I.A.**

Universidad de Granada



# Patrones en árboles

© Fernando Berzal, [berzal@acm.org](mailto:berzal@acm.org)

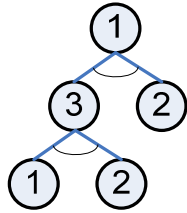
# Patrones en árboles



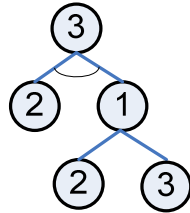
- Tipos de árboles
- Tipos de subárboles / patrones en árboles
- POTMiner [Partially-Ordered-Tree Miner]
- Algoritmos
- Aplicaciones



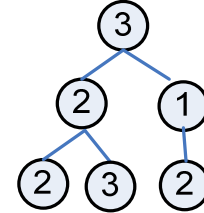
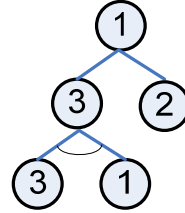
# Tipos de árboles



Árbol ordenado



Árboles parcialmente ordenados



Árbol no ordenado

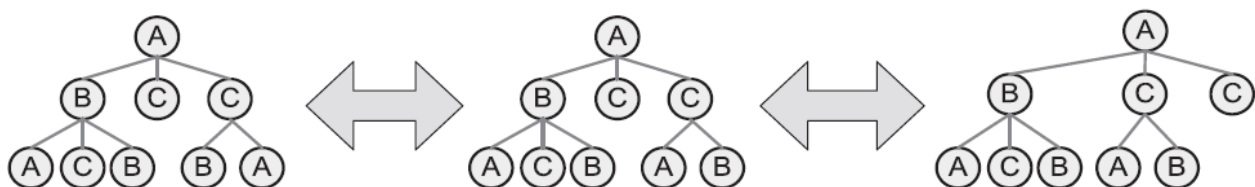


# Tipos de árboles



## Ejemplo

3 representaciones alternativas del mismo árbol [no ordenado]



### Codificación del árbol

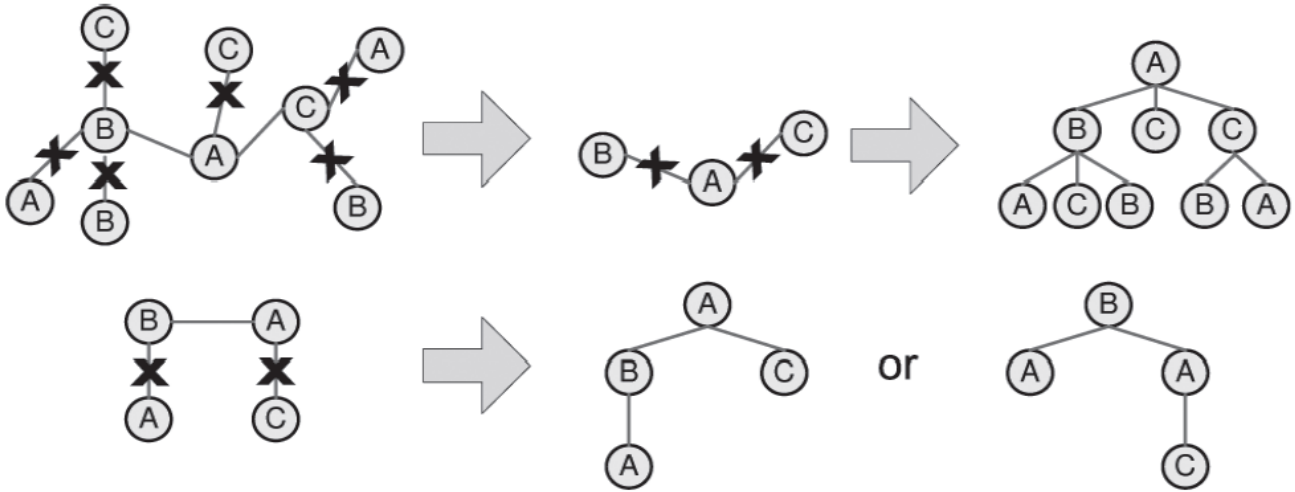
- DFS [Depth-first]: A C B ↑ A ↑ ↑ B
- BFS [Breadth-first]: A \$ C B \$ B A
- Depth sequence: (0,A) (1,C) (2,B) (2,A) (1,B)



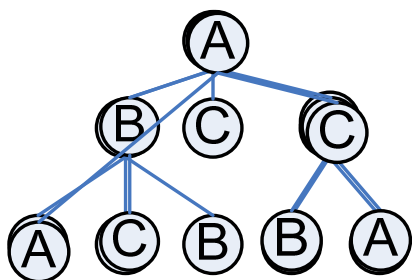
# Tipos de árboles



**“Free trees”** (sin raíz asignada)



# Tipos de subárboles

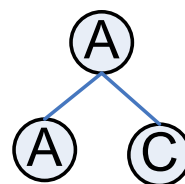


Original tree

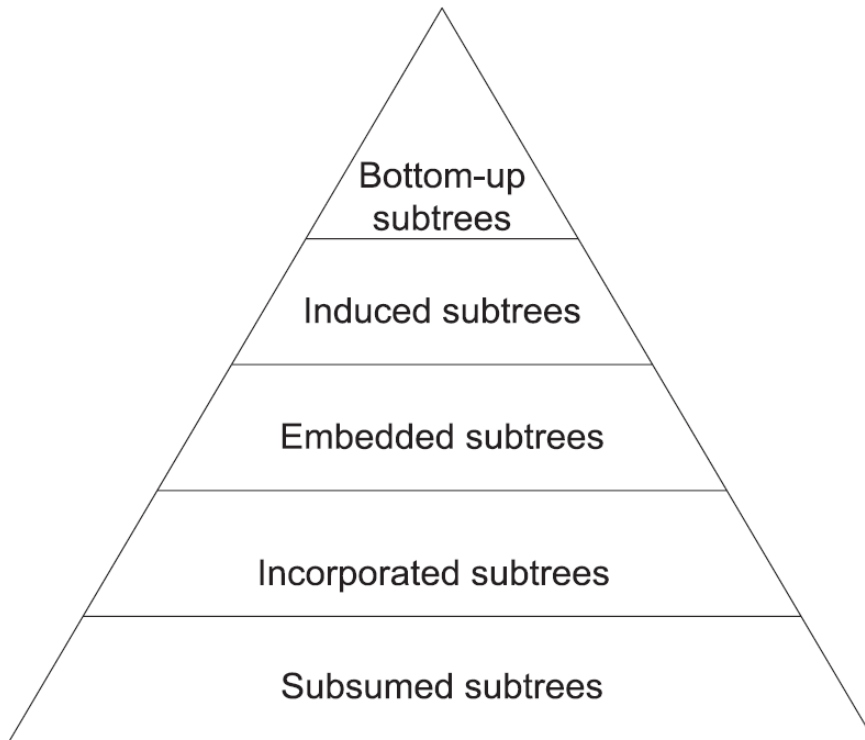
Bottom-up subtree

Induced subtree

Embedded subtree



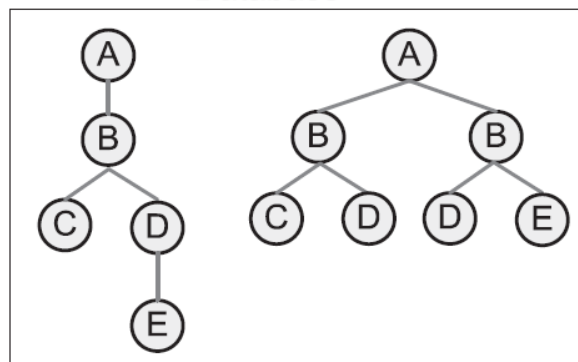
# Tipos de subárboles



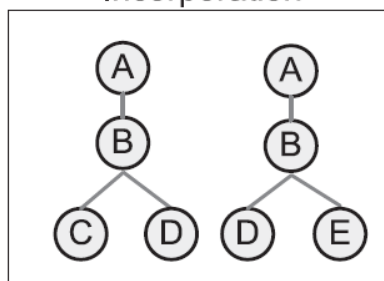
# Tipos de subárboles



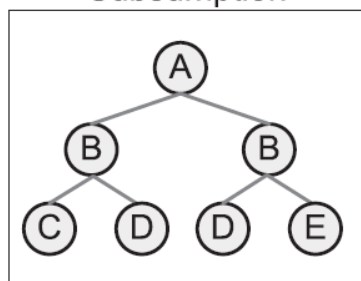
Database



Incorporation



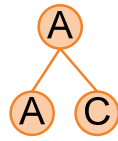
Subsumption



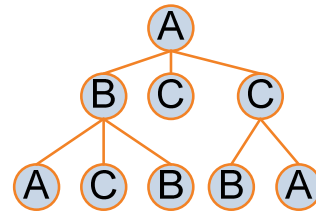
# Tipos de subárboles



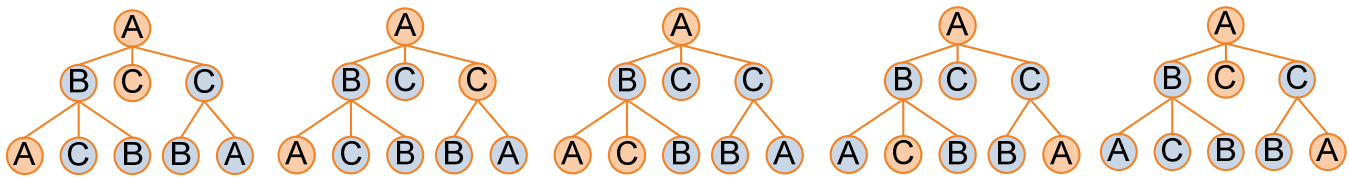
## Embedded subtree



## Original tree



Ocurrencias del patrón en el árbol original



# POTMiner

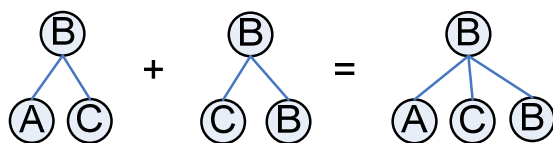


## Partially-Ordered Tree Miner

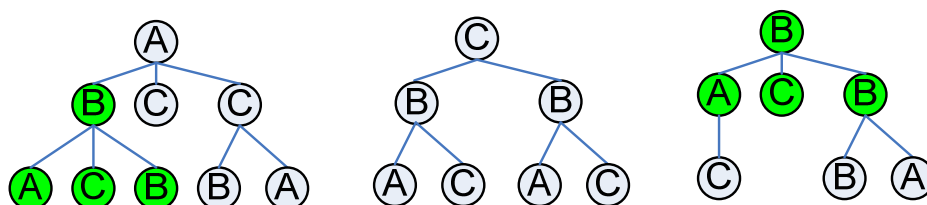
Jiménez, Berzal & Cubero (KAIS'2009)

Algoritmo tipo Apriori:

- Generación de candidatos

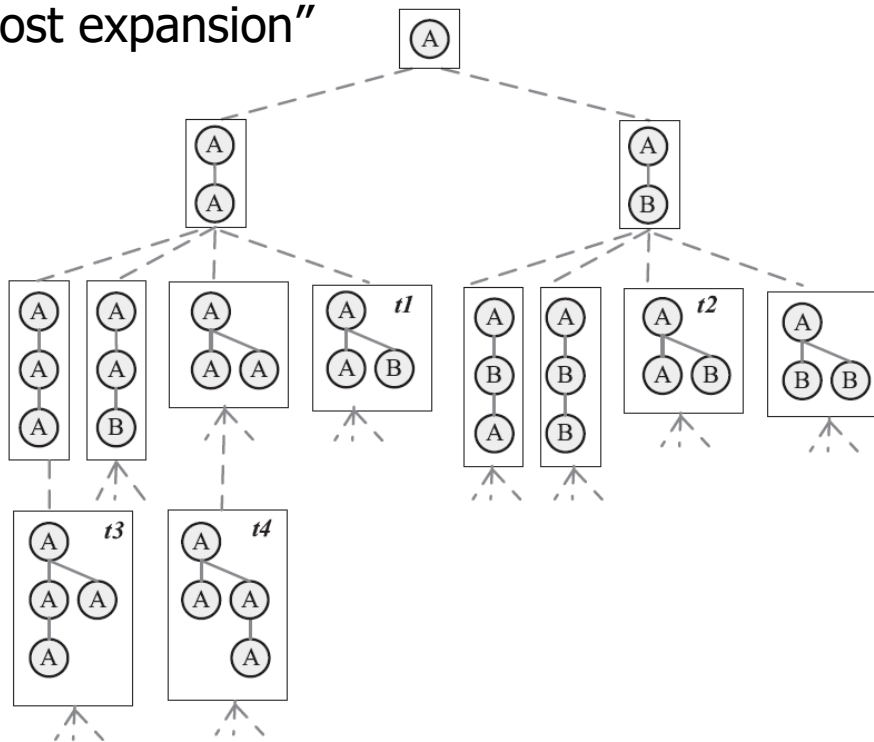


- Conteo del soporte



## Generación de candidatos

“Rightmost expansion”



## Generación de candidatos

Diferentes estrategias

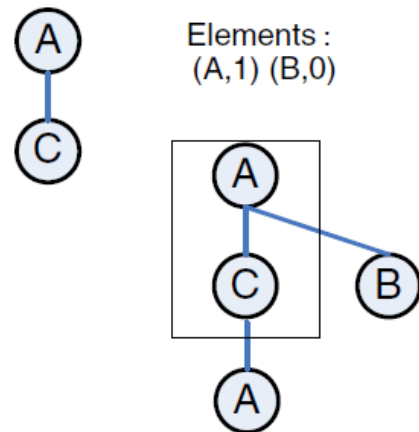
- “Rightmost expansion” (FreqT)
  - TMG [Tree-Model-Guided] candidate enumeration
  - ... con secuencias de profundidad (Unot, uFreqt, Gaston, TRIPS)
- Extensión basada en clases de equivalencia (TreeMiner, SLEUTH, POTMiner, RETRO, Phylominer)
- “Right-and-left” tree join (AMIOT)
- “Extension and join” (HybridTreeMiner)







## Clases de equivalencia

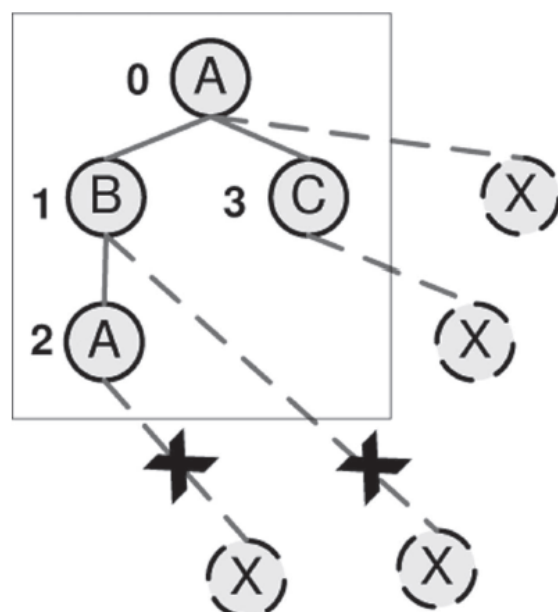


Clase de equivalencia  
con dos elementos,  $ACA$  y  $AC\uparrow B$   
que comparten el prefijo  $AC$



## Generación de candidatos

Clases de equivalencia



IDEA

Generar todos los patrones  
posibles, pero sin generar  
patrones por duplicado...

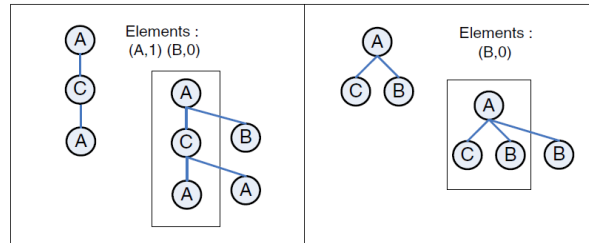




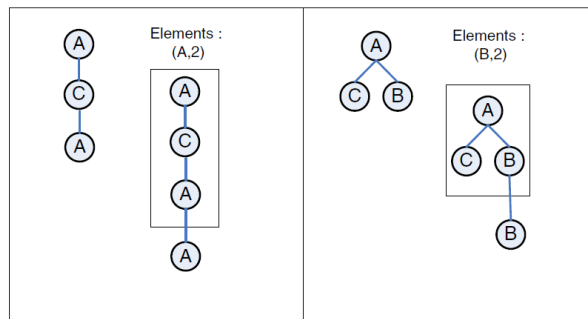
## Generación de candidatos

Clases de equivalencia

- "Cousin extension" (en anchura)

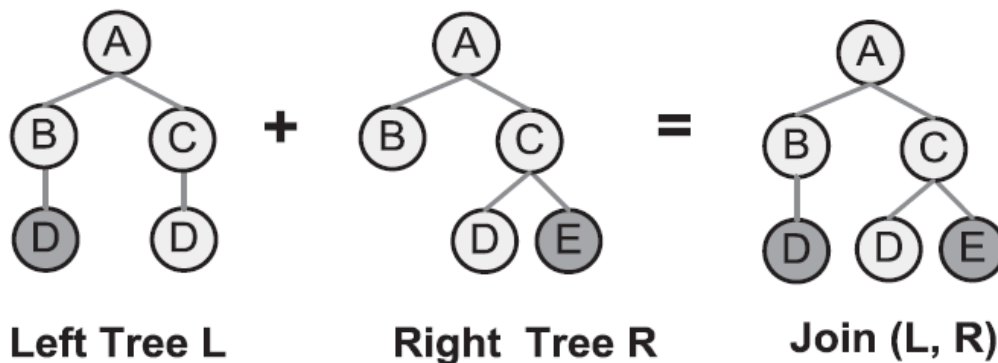


- "Child extension" (en profundidad)



## Generación de candidatos

"RL Tree Join"

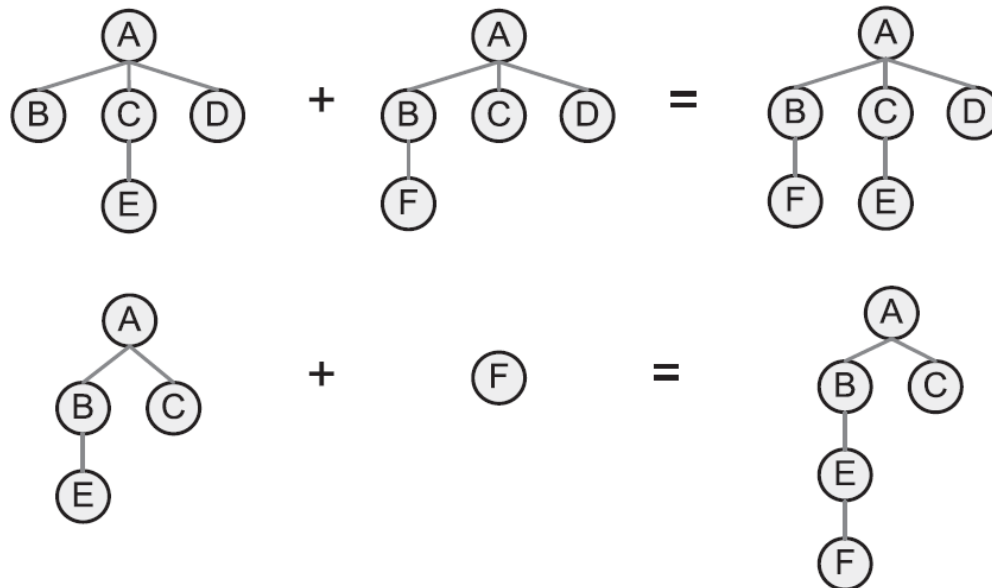




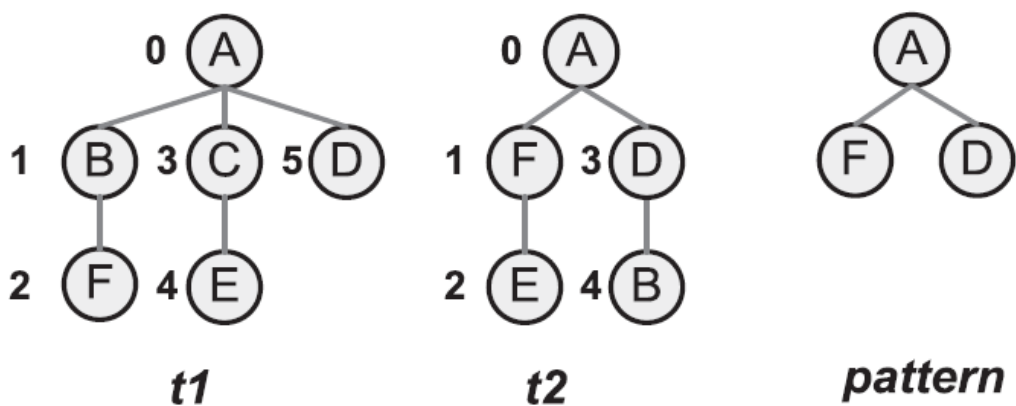


## Generación de candidatos

"Extension and join"



## Cálculo del soporte



■ Listas de ocurrencias **(tid, i<sub>1</sub>, i<sub>2</sub>... i<sub>k</sub>)**

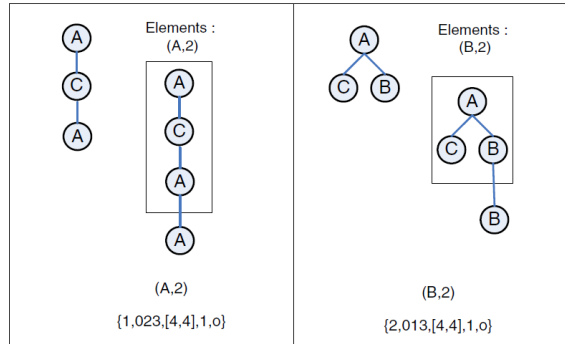
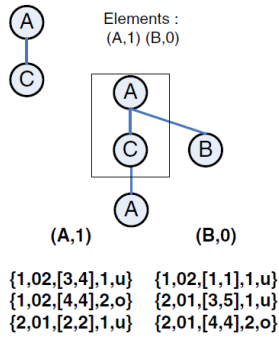
■ Listas de ámbitos **(tid, m, s)**



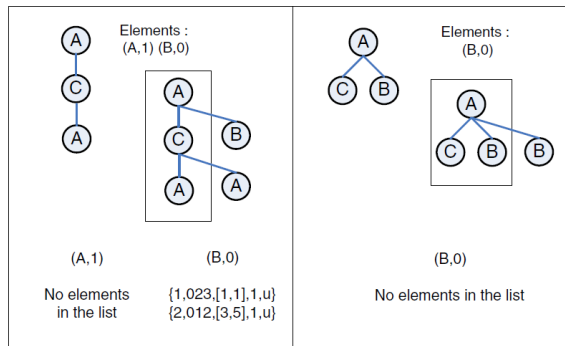


## Cálculo del soporte

### Reunión de listas de ámbitos



In-scope join  
(child extension)

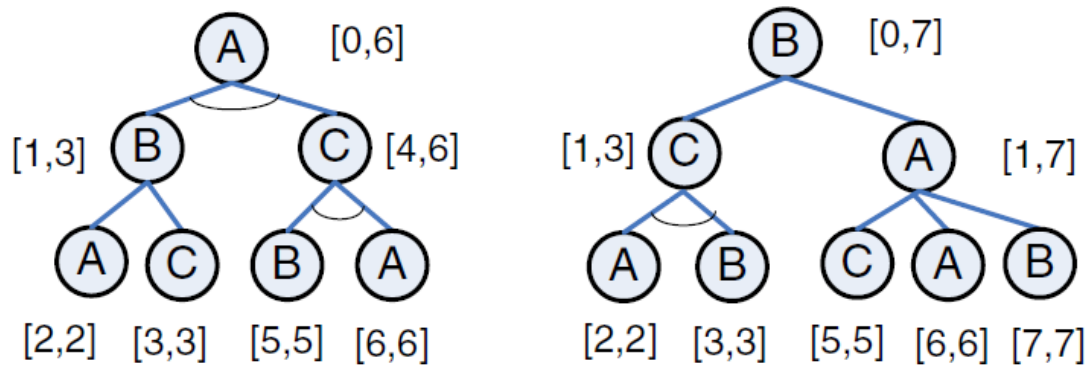


Out-scope join  
(cousin extension)



## Ejemplo

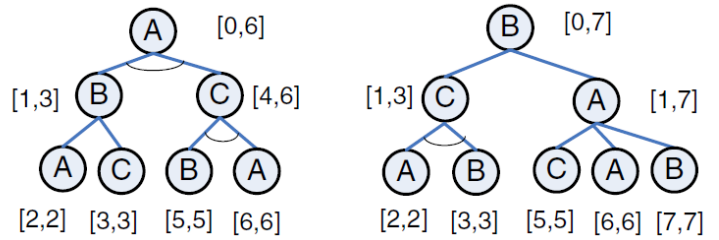
### Conjunto de datos





## Ejemplo

Patrones de tamaño 1  
("representación vertical")



- {1,\_, [0,6], 0, \_}
- {1,\_, [2,2], 2, u}
- {1,\_, [6,6], 2, o}
- {2,\_, [2,2], 2, o}
- {2,\_, [1,7], 1, u}
- {2,\_, [6,6], 2, u}

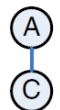
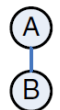
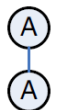
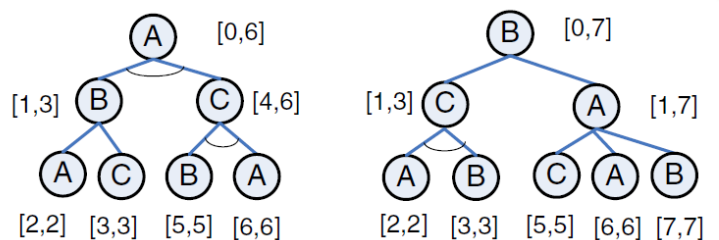
- {1,\_, [1,3], 1, \_}
- {1,\_, [5,5], 2, o}
- {2,\_, [0,7], 0, \_}
- {2,\_, [3,3], 2, o}
- {2,\_, [7,7], 2, u}

- {1,\_, [4,6], 1, o}
- {1,\_, [3,3], 2, u}
- {2,\_, [1,3], 1, u}
- {2,\_, [5,5], 2, u}

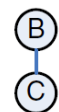
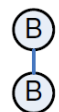
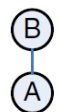


## Ejemplo

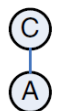
Clases de equivalencia  
derivadas de los patrones  
de tamaño 1



- {1,0,[2,2],2,u}
- {1,0,[6,6],2,o}
- {2,4,[6,6],2,u}
- {1,0,[1,3],1,o}
- {1,0,[5,5],2,o}
- {2,4,[7,7],1,u}
- {1,0,[4,6],1,o}
- {1,0,[3,3],2,u}
- {2,4,[5,5],1,u}



- {1,1,[2,2],1,u}
- {2,0,[3,3],2,o}
- {1,1,[3,3],1,u}
- {2,0,[4,7],1,u}
- {2,0,[7,7],2,u}
- {2,0,[1,3],1,u}
- {2,0,[6,6],2,u}
- {2,0,[5,5],2,u}



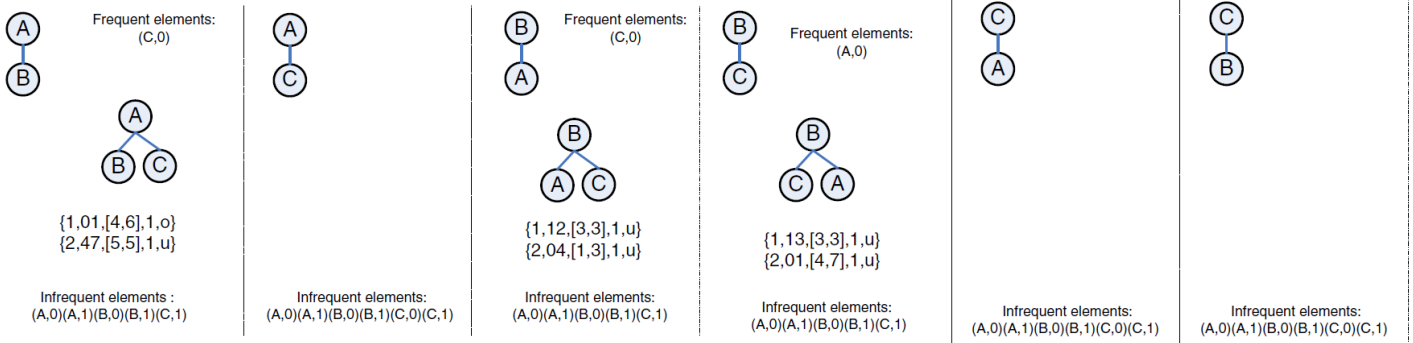
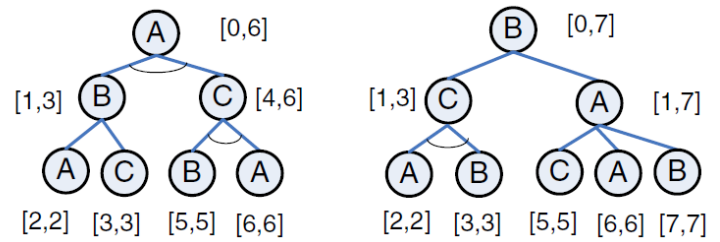
- {1,4,[6,6],1,o}
- {2,1,[2,2],1,o}
- {1,4,[5,5],1,o}
- {2,1,[3,3],1,o}
- No elements in the scope list





## Ejemplo

Clases de equivalencia derivadas de los patrones de tamaño 2



Algoritmo basado en clases de equivalencia (como SPADE para secuencias, TreeMiner/Sleuth para árboles)

algorithm *POTMiner*

Obtain frequent nodes (frequent patterns of size 1)

Build candidate classes  $C_1$  from the frequent nodes

for  $k=2$  to MaxSize

  for each class  $P \in C_{k-1}$

    for each element  $p \in P$ .

      Compute the frequency of  $p$

      if  $p$  is frequent

      then

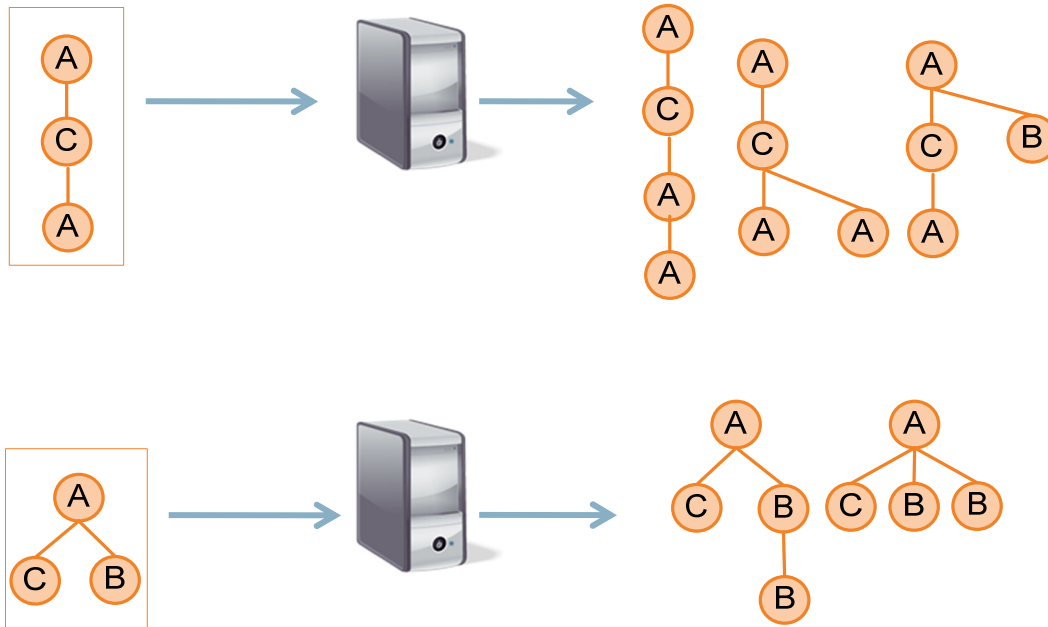
        Create a new class  $P'$  from  $p$ .

        Add  $P'$  to  $C_k$



## Implementación paralela

Distribución de candidatos [CD: Candidate Distribution]



## Implementación paralela

**algorithm** *ParallelPOTMiner*

Obtain frequent nodes (frequent patterns of size 1)

Build candidate classes  $C_1$  from the frequent nodes

for  $k=2$  to MaxSize

  for each class  $P \in C_{k-1}$

    Extend  $P$  in parallel to obtain  $P_{extended}$

  for each class  $P \in C_{k-1}$

$C_k = C_k \cup P_{extended}$

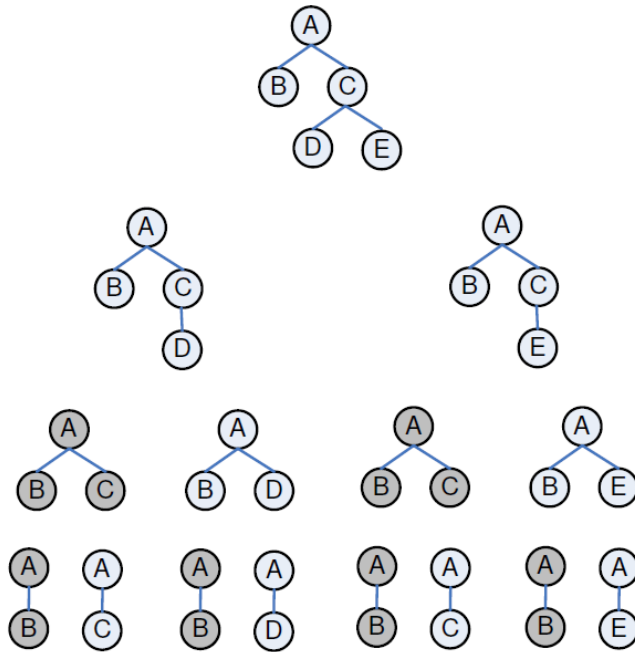






## POTMiner "light"

(generación de listas de ámbitos bajo demanda)



algorithm *scopeList* (Tree  $t$ ):  $s$   
 //  $t : n_1, n_2..n_{k-1}, n_k$

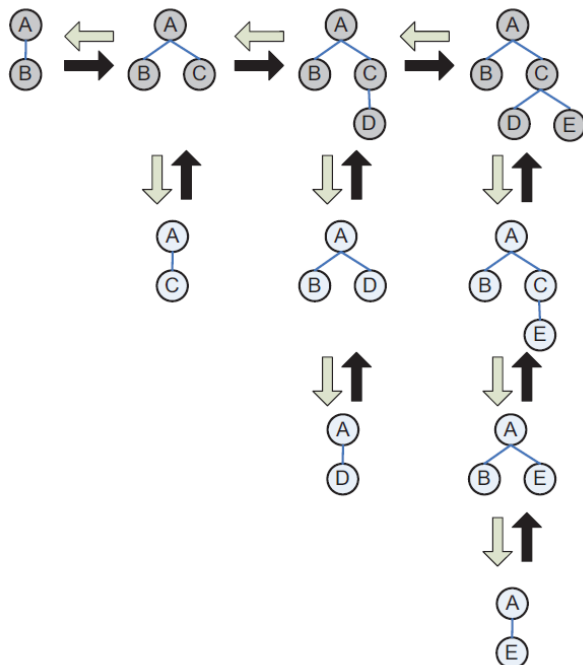
```

if  $k = 1$ 
then
     $s =$  scope list of node  $n_k$ 
else
     $t_1 = t - n_k$ 
     $t_2 = t - n_{k-1}$ 
     $s_1 = scopeList(t_1)$ 
     $s_2 = scopeList(t_2)$ 
    if  $n_k.parent = n_{k-1}$ 
    then
        //  $t$  was obtained by child extension
         $s = in-scope-join(s_1, s_2)$ 
    else
        //  $t$  was obtained by cousin extension
         $s = out-scope-join(s_1, s_2)$ 
    
```



## POTMiner DP ["dynamic programming"]

Tiempo de CPU vs. Uso de memoria



algorithm *scopeListDP* (Tree  $t$ ):  $s$   
 //  $t : n_1, n_2..n_{k-1}, n_k$

```

for  $i=1$  to  $k$ 
     $list[i] =$  scope list of node  $n_i$ 
    for  $j=1$  to  $i-1$ 
        if  $j=1$ 
            then // pattern of size 2
                 $list[i] = in-scope-join(list[j], list[i])$ 
            else
                 $s = subtree[j+1][i]$  //  $s : s_1, s_2..s_j, s_{j+1}$ 
                if  $s_{j+1}.parent = s_j$ 
                then
                     $list[i] = in-scope-join(list[j], list[i])$ 
                else
                     $list[i] = out-scope-join(list[j], list[i])$ 
    return  $list[k]$ 
    
```







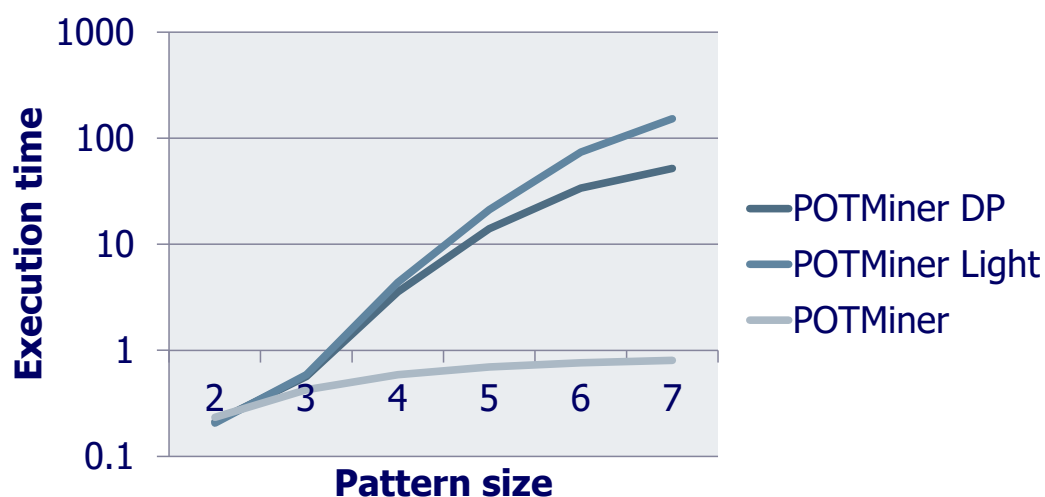
## Eficiencia

Algorithm	CPU Time	Memory consumption (number of scope lists)
POTMiner	$O\left(\frac{t * (Ln^2)^{MaxSize}}{(MaxSize - 1)!}\right)$	$O(L^{MaxSize-1} * (MaxSize - 2)!)$
POTMiner Light	$O\left(\frac{t * (2Ln^2)^{MaxSize}}{(MaxSize - 1)!}\right)$	$O(L)$
POTMiner DP	$O\left(\frac{t * MaxSize^2 * (Ln^2)^{Maxsize}}{(Maxsize - 1)!}\right)$	$O(L + k - 1)$



## Resultados experimentales

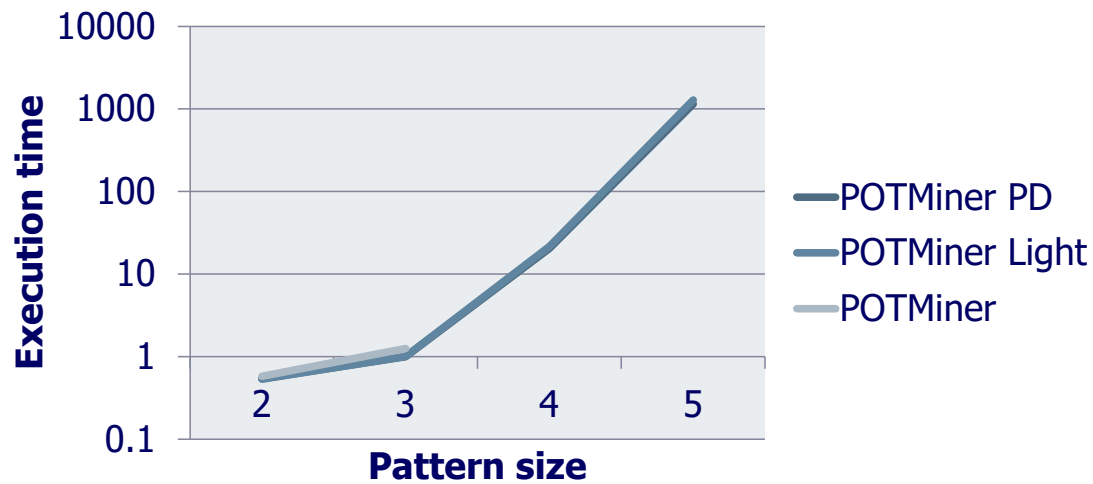
Datos sintéticos





## Resultados experimentales

Datos reales



## Algoritmos



Algorithm	Input trees				Identified patterns			
	Ordered trees	Partially -ordered	Unordered trees	Free trees	Induced subtrees	Embedded subtrees	Incorporated /Subsumed	Maximal /Closed
FreqT [1]	•				•			
AMIOT [15]	•				•			
uFreqT [21]			•		•			
HybridTreeMiner [11]			•	•	•			
Unot [4]			•		•			
FreeTreeMiner [10]			•	•	•			
FreeTreeMiner' [25]			•	•	•			
GASTON [22]			•	•	•			
X3Miner [26]	•					•		
MB3Miner [27]	•					•		
IMB3Miner [28]	•					•		
TreeMiner [38]	•					•		
TreeMinerD [38]	•					•		
RETRO [7]	•				•	•	•	
Chopper [34]	•					•		
XSpanner [34]	•					•		
Uni3 [13]			•			•		
Phylominer [41]			•			•		
SLEUTH [37]			•			•		
POTMiner [16]	•	•	•		•	•		
TRIPS [29]	•		•		•	•		
TIDES [29]	•		•		•	•		
CMTreeMiner [9]	•		•		•			•
PathJoin [35]			•		•			•
DRYADE [31]			•			•		•
TreeFinder [30]			•				•	•





Algorithm	Tree representation	Candidate generation approach	Implementation details
FreqT [1]	–	Rightmost expansion	RMO occurrence lists
AMIOT [15]	–	Right and left union	RMO occurrence lists
uFreqT [21]	Depth sequences	Rightmost expansion with depth sequences	Bipartite graphs
HybridTreeMiner [11]	Breadth-first codification	Union-extension method	Occurrence lists
FreeTreeMiner [10]	Depth-first codification	Apriori itemset generation	Indexation techniques
FreeTreeMiner* [25]	Depth-first codification	Maximal-depth extension	
TreeMiner [38]	Depth-first codification	Equivalence classes	Scope lists
TreeMinerD [38]	Depth-first codification	Equivalence classes	Scope lists for non-weighted support
RETRO [7]	Relational representation	Equivalence classes	Scope lists
Chopper [34]	Depth sequences	N/A	Frequent subsequences (PrefixSpan [23])
X3Miner [26]	Depth-first codification	Rightmost expansion – TMG enumeration	Vertical occurrence lists
MB3Miner [27]	Depth-first codification	Rightmost expansion – TMG enumeration	Vertical occurrence lists
IMB3Miner [28]	Depth-first codification	Rightmost expansion – TMG enumeration	Vertical occurrence lists
XSpanner [34]	Depth sequences	N/A	Frequent subsequences (PrefixSpan [23])
SLEUTH [37]	Depth-first codification	Equivalence classes	Scope lists
Unot [4]	Depth sequences	Rightmost expansion with depth sequences	Occurrence lists
Phylominer [41]	Depth-first codification	Equivalence classes	No labels in internal nodes
Uni3 [13]	Depth-first codification	Rightmost expansion – TMG enumeration	Vertical occurrence lists
GASTON [22]	Depth sequences	Rightmost expansion with depth sequences	
TRIPS [29]	Post-order codification	Embedding lists	Support Structure (hash table)
TIDES [29]	Depth sequences	Rightmost expansion with depth sequences	Support Structure (hash table)
POTMiner [16]	Depth-first codification	Equivalence classes	Scope lists
CMTreeMiner [9]	Depth-first codification	N/A	DAG enumeration graph
TreeFinder [30]	Relational representation	Apriori itemset generation	Clustering techniques
PathJoin [35]	FST-Forest structure	N/A	
DRYADE [31]	Propositional language	–	External closed itemset miner

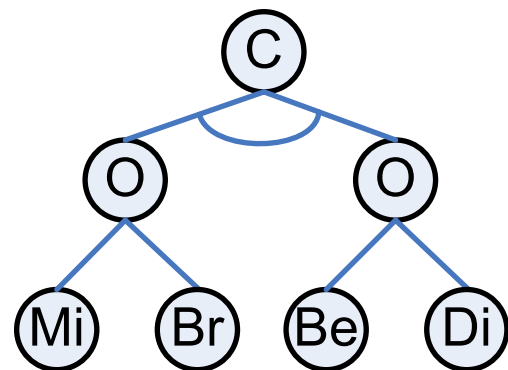


## Documentos XML

```

<customer>
  <order>
    <item>milk</item>
    <item>bread</item>
  </order>
  <order>
    <item>beer</item>
    <item>diapers</item>
  </order>
</customer>

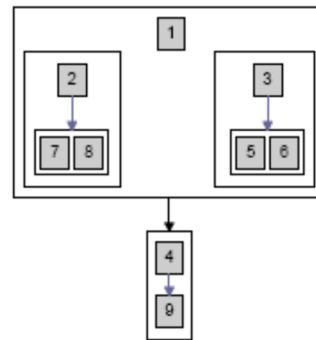
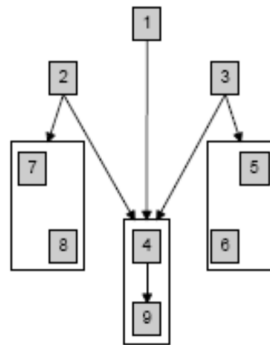
```



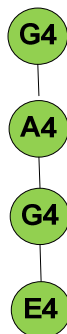


## Análisis de software (grafos de dependencias)

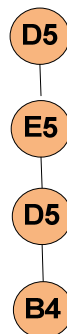
```
public int TestNestedFor (int n) {  
    int sum = 0; 1  
    for (int i=0; 2 i<n; 6 i++) 7  
        for (int j=0; 3 j<n; 6 j++) 5  
            sum += 1; 4  
    return sum; 9  
}
```



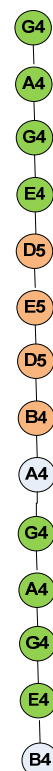
## Patrones musicales



Patrón exacto



Patrón similarn



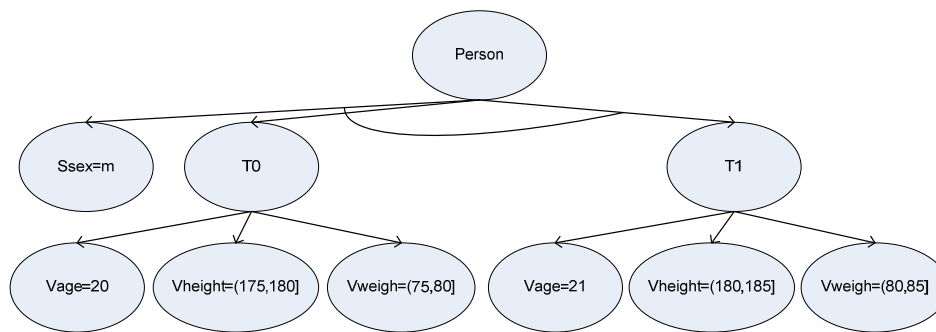


## Estudios longitudinales

### Representación basada en el tiempo

Person							
id	Ssex	V0age	V0height	V0weigh	V1age	V1height	V1weigh
1	m	20	(175,180]cm	(75,80]kg	21	(180,185]cm	(80,85]kg
2	f	16	(155,160]cm	(50,55]kg	17	(160,165]cm	(60,65]kg

Person				
id	Ssex	Vage	Vheight	Vweigh
1	m	20	(175,180]cm	(75,80]kg
1	m	21	(180,185]cm	(80,85]kg
2	f	16	(155,160]cm	(50,55]kg
2	f	17	(160,165]cm	(60,65]kg

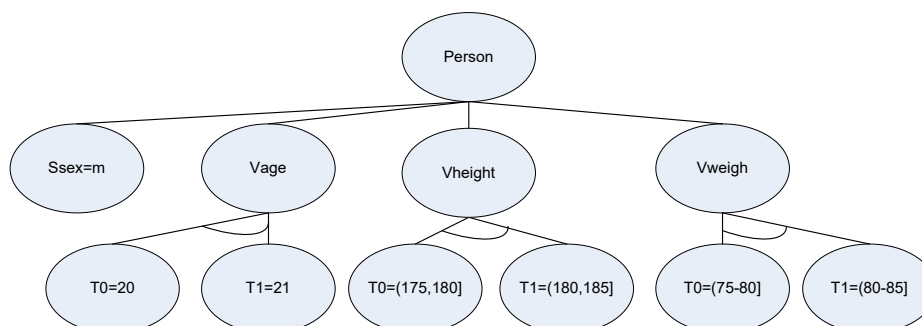


## Estudios longitudinales

### Representación basada en variables

Person							
id	Ssex	V0age	V0height	V0weigh	V1age	V1height	V1weigh
1	m	20	(175,180]cm	(75,80]kg	21	(180,185]cm	(80,85]kg
2	f	16	(155,160]cm	(50,55]kg	17	(160,165]cm	(60,65]kg

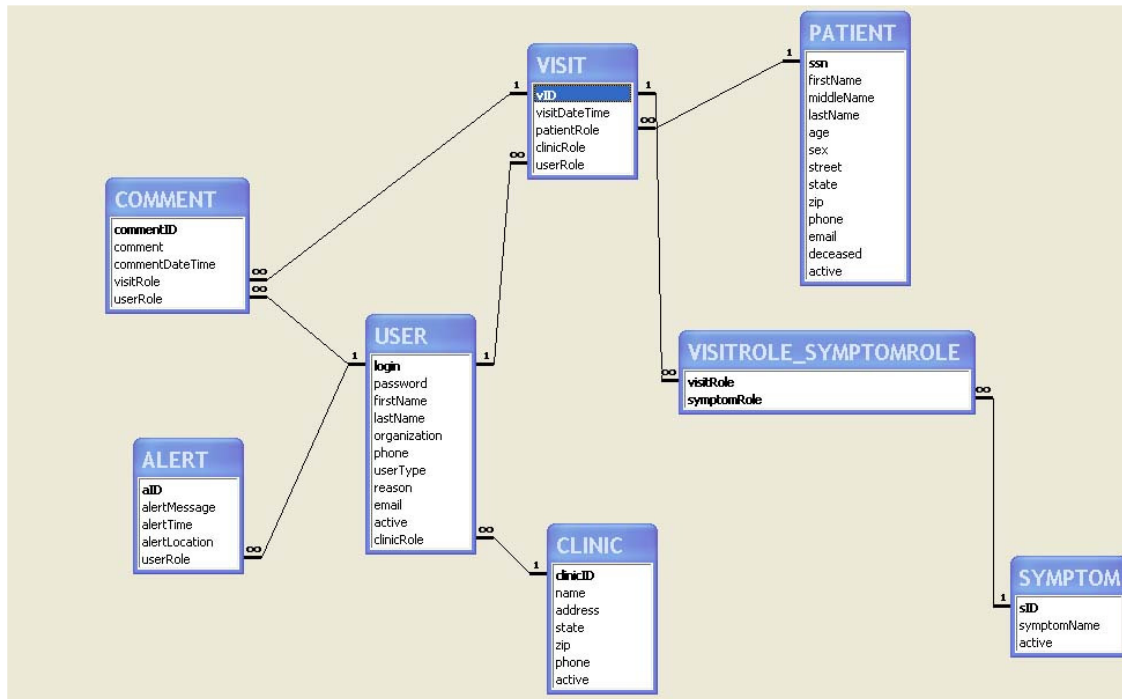
Person				
id	Ssex	Vage	Vheight	Vweigh
1	m	20	(175,180]cm	(75,80]kg
1	m	21	(180,185]cm	(80,85]kg
2	f	16	(155,160]cm	(50,55]kg
2	f	17	(160,165]cm	(60,65]kg



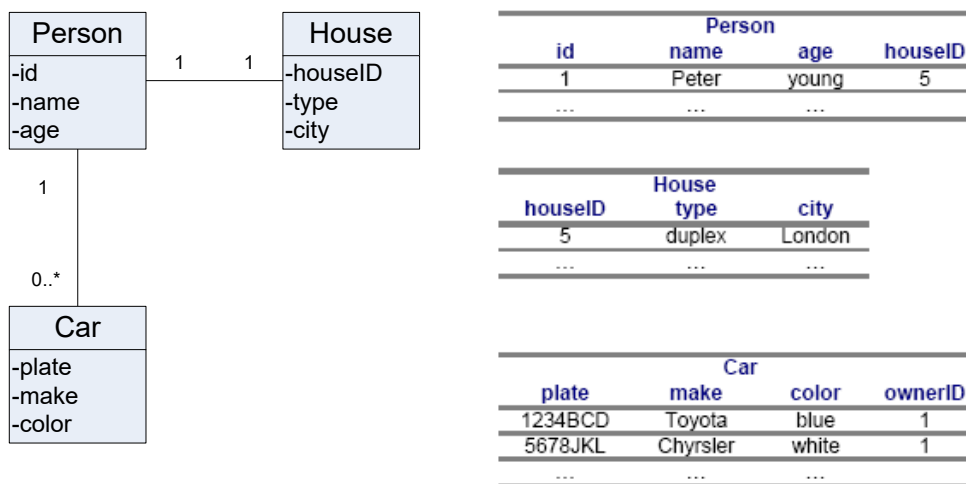




## Bases de datos multirelacionales

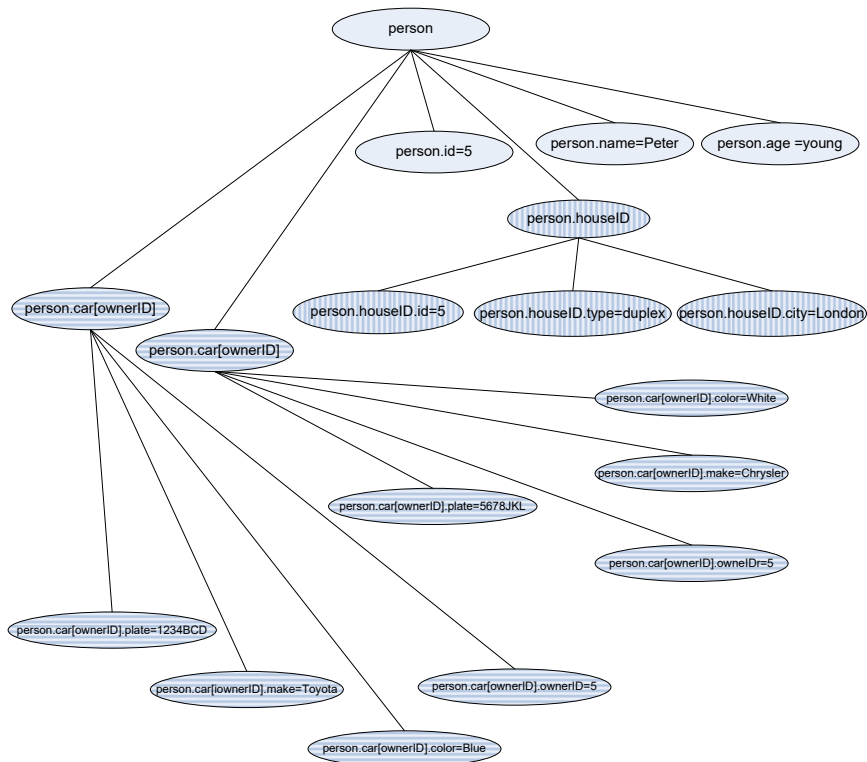


## Bases de datos multirelacionales





## Bases de datos multirelacionales



## Bases de datos multirelacionales

Dos tipos de patrones

- E = Embedded subtrees
- I = Induced subtrees

Dos esquemas de representación

- K = "Key-based tree representation"
- O = "Object-based tree representation"

$$IK \subseteq EK \subset_{eq} IO \subseteq EO$$



# Agradecimientos



**Aída Jiménez, Ph.D.**

Tesis doctoral

**Knowledge discovery in non-linear structures**

Departamento de Ciencias de la Computación e I.A.

Marzo de 2011

“If you torture the data long enough,  
it will confess” -- Ronald Coase



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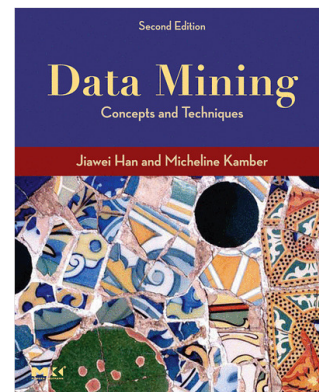
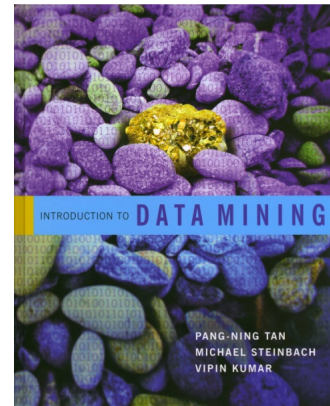
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